PHASOR SIGNAL ANALYSIS OF THE SIX-PORT

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ABSTRACT

This paper presents a detailed mathematical and physical representation of phasor signals in the six-port device. The interpretation and analysis provides a clear insight and precise knowledge of how and why the six-port actually works. A composite diagram is presented which makes it possible to select the design parameters and illustrates the interrelationships of six-port parameters.

Introduction

In the past there has not been a concerted attempt to present a clear and detailed descriptive insight into the six-port operational characteristics, how and why it works, and what the specific design requirements are. This paper describes six-port equations in terms of the corresponding phasor signals.

The basic phasor signal analysis set forth in this paper is applicable to both single-port and dual six-port systems when considered strictly in terms of the phasor signal reltaionships.

The Six-Port

A six-port device is illustrated in Figure 1. It is described by the following equations:

$$b_4 = Ca_2 + Db_2 \quad (1)$$

$$b_3 = Aa_2 + Bb_2 \quad (2)$$

$$b_5 = Ea_2 + Fb_2 \quad (3)$$

$$b_6 = Ga_2 + Hb_2 \quad (4)$$

where b_3, b_5 and b_6 are measured signals which are proportional to the signals a_2 and b_2. A - H are parameters of the six-port which are obtained by system calibration. These constants and the measured parameters are used to calculate a_2 and b_2 using the four linear equations.

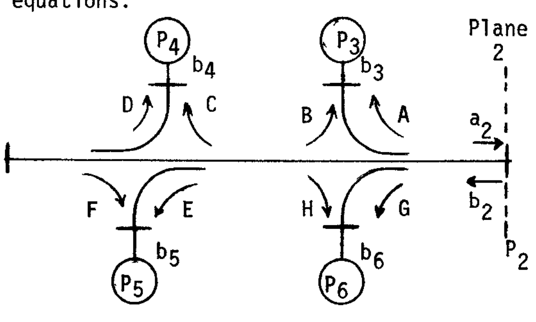

FIGURE 1. Illustration of a Six-Port.

Assume a matched source and B, F, and H are zero. A short circuit at the reference plane will result in certain power levels at the P_3, P_5 and P_6 meters. If the short is replaced with a reflection coefficient having a return loss of 20 dB, the three power meter readings will decrease an amount corresponding to 20 dB provided each is an accurate measuring device. However, the established power levels will be constant regardless of the phase of the short or the termination. Measurement of phase of the reflection coefficient is possible only if there are signals through the paths B, F, and H.

There are two possible conditions that can exist at the detectors. The signals Bb_2, Fb_2 and Hb_2 can be either greater than or smaller than the corresponding Aa_2, Ea_2 and Ga_2 signals. These conditions are illustrated in Figure 2, assuming again that there is no source mismatch looking back into the system. The phasor signal relationships and the measurements in dB are illustrated on the same diagram as a matter of convenience.

Phasor Relationships

Figure 2 illustrates the **phasor** relationships for the port defined by Equation 2. K, K', J and J' are the relative attenuation (dB) values corresponding to the in-phase and out-of-phase conditions as illustrated. S represents the total variation between the in-phase and out-of-phase conditions expressed in dB. M and N represent the dB measurement from the zero return loss reference level to the in-phase condition as shown. The -U and +U value represent the return loss, in dB, between the two signals for the conditions shown in Figure 2a and b and as indicated on the graph of Figure 3. Figure 2a represents the condition of Bb_2 greater than Aa_2. The calculations for the dB equivalent of Bb_2 and the Return Loss (R_L) which corresponds to the reflection coefficient Γ_ℓ are:

$$D_B = M + K \quad (5)$$

$$R_L = M + J \quad (6)$$

Figures 2b and c indicate the condition when Aa_2 is greater than Bb_2 and this represents a direct measurement of the return loss corresponding to the reflection coefficient, i.e., E_r greater than Bb_2. The dB equivalents of the two signals are:

$$D_B = N + J \quad (7)$$

$$R_L = N + K \quad (8)$$

The magnitude of the reflection coefficient is calculated as

$$|\Gamma_\ell| = 10^{-R_L/20} \quad (9)$$

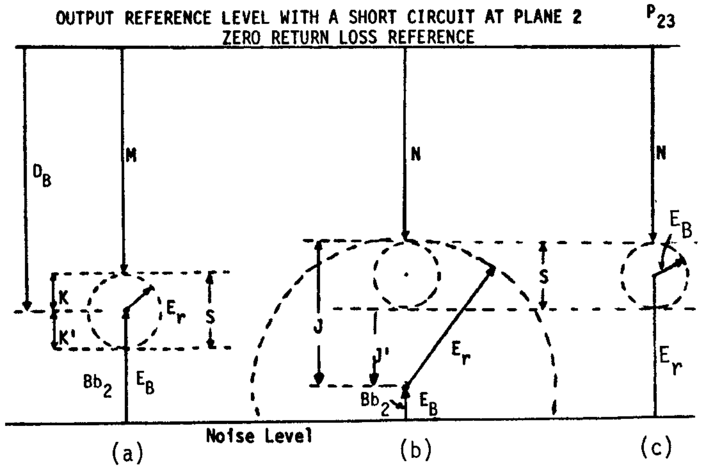

FIGURE 2. (a) The signal Bb_2 is greater than the signal E_r which represents the reflection coefficient. (b) The actual relationships involved in the calculation of the smaller signal (E_r). (c) Illustrates that the output meter always sees the larger signal as in (a).

Figure 3 illustrates the phasor relationships of any two signals. The left side represents the measurement of Bb_2 as the larger signal. Therefore, the reflection coefficient is the smaller signal and must be calculated according to the correction of Equation 2, as previously set forth. If the reflection coefficient signal is larger than the forward coupled signal Bb_2, the representation is the right side of Figure 3 and Figure 2b and c with the corresponding mathematics of Equation 4.

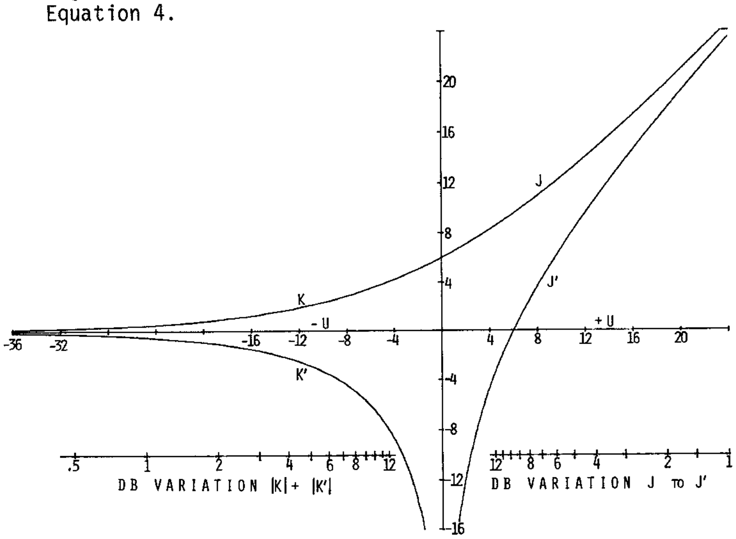

FIGURE 3. The Universal Error Function which defines the phasor relationships of any two signals.

Six-Port Parameters

The single six-port measures the complex reflection coefficient using amplitude measurements at these detectors which have certain relative phase conditions. The unit circle, the q points, and their ideal phase relationships are illustrated in Figure 4 as set forth by Engen.[5,6,8,10]

Assume the literal definition of the unit circle as a representation of $\Gamma = 1$. This represents the condition where $Aa_2 = Bb_2$ with a short circuit connec- at the measurement reference Plane 2. Figure 3 illustrates that if -B/A is close to unity, the reflection coefficient of 1.0 cannot be measured. Also, a large signal Bb_2 presents a major problem in the measurement of small values of reflection coefficient due to the very large difference in the magnitudes of the two signals at each detector which is required to measure phase as previously pointed out. Accurate measurements would require extremely accurate measurement of very small signal magnitudes.

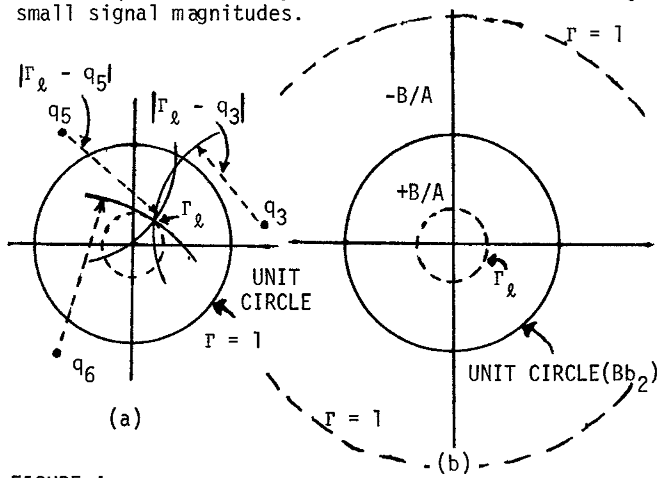

FIGURE 4. Illustration of q points and the unit circle. (a) Ideal relationships described by Engen. (b) The general theory.

Better understanding and better design of six-ports are possible by eliminating the literal definition of the $\Gamma = 1$ unit circle and expanding the definition in terms of a general theory as follows.

The unit circle is the circle that represents the condition where the forward coupled signal and the signal which represents the magnitude of the reflection coefficient have the same amplitude at the particular detector, i.e., where $Aa_2 = Bb_2$, $Ea_2 = Fb_2$ and $Ga_2 = Hb_2$. In physical systems, each port has its own unit circle. The unit circle is represented by the Y-axis at U = Zero in Figure 3. Since the two signals are equal, the possible dB variation would be from 6.02 dB to minus infinity -- a well known fact. The graph of Figure 3 indicates that the system cannot measure the variation which results when the two signals approach the same value. This can result in "holes" in the calibration and/or measurement processes unless one uses only the continuous functions K and J.

The q points define the measured value of reflection coefficient which results in the unit circle. The -B/A, -F/E, and -H/G values represent reflection coefficient values outside the unit circle and +B/A, +F/E, and +H/G represents reflection coefficient values inside the unit circle.

Calibration and Measurement

Measurement and/or calibration conditions for a single six-port are illustrated in Figure 5. The power level P_4 is completely independent of the reflectometer measurement measurement parameters and is, therefore, only used as a constant reference level during system calibration and measurement. If C is not equal to zero, there is the added complexity of accounting for a non-constant P_4 (indicated by the dotted circle around P_4) which results when the short circuit is varied in phase at the reference plane.

Also, the source mismatch causes a variation in the power levels at each detector, as illustrated by the dotted circle on the P_{23} zero return loss reference line. There is a measurement technique which can be used in the calibration process to establish the amplitude and phase of this correction at the reference plane.

Figure 5 illustrates that the measurement technique can establish the reflection coefficient without establishing A, B, a_2 and b_2 separately. It, therefore, appears that the calibration and measurement techniques can be somewhat simplified by the direct measurements of B, F, H and Γ_ℓ.

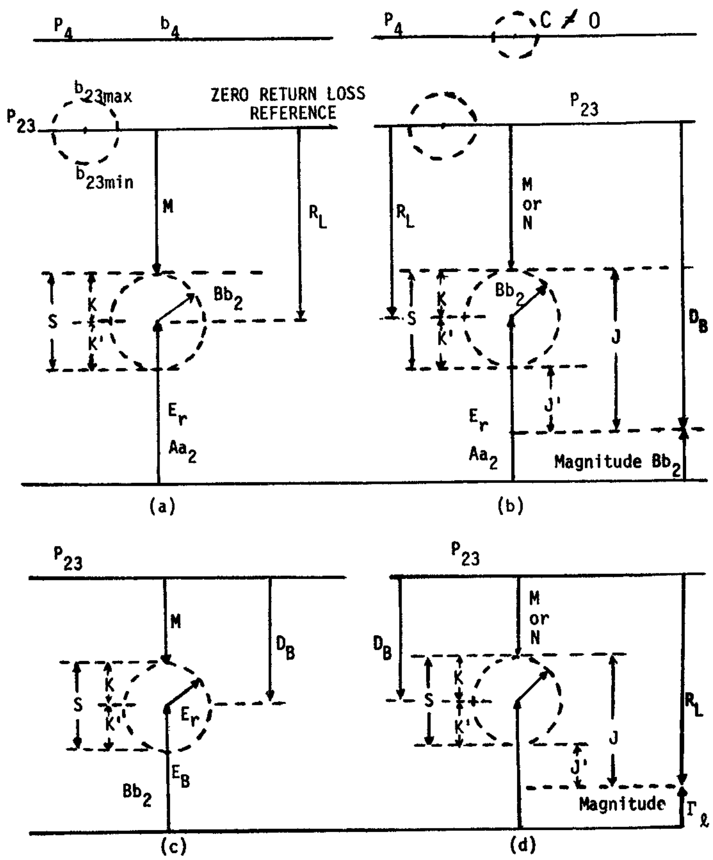

FIGURE 5. (a) and (b) Signal relationships when the termination standard produces a reflection that is greater than Bb_2. (c) and (d) the termination standard produces a reflection smaller than Bb_2.

Long-line effects can occur in the practical six-ports if the phase relationships are not maintained. Differences in electrical lengths of the line detectors for the paths of Bb_2, Fb_2 and Hb_2 or Aa_2, Ea_2 and Ga_2 will result in rotation of the q points as a function of frequency. As an example, the phase relationships for q_3 and q_6 of Figure 1 might be maintained, but q_5 would rotate as a function of frequency and the difference in electrical length.

Conclusions

This paper makes it quite clear that the six-port developer should devise the calibration and measurement processes so that the relative amplitudes of the various parameters are known. It also shows that one can definitely establish all relationships during the calibration process so that the system characteristics and the characteristics of the standards are precisely established.

A thorough understanding of the relationships pointed out here should considerably simplify the development of better six-ports for specific applications. We feel that this will also lead to a much better understanding of, and a greater appreciation for, the rigorous mathematics which produce the six-port concepts.

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